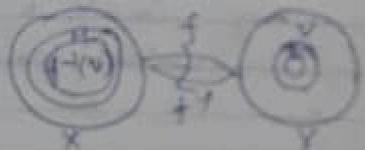
4/5/2020 Wednesday (13)

Thm. The function $f:(X,\mathfrak{I}) \to (Y,\mathcal{U})$ is continuous iff $f^{-1}(V)$ is open in X for every open set V in Y.

Proof:- Let $f:(X,\mathfrak{I}) \to (Y,\mathcal{U})$ be a map
suppose f is continuous. Let V be an open set of Y.
To prove that $f^{-1}(V)$ is open in X
If $f^{-1}(V) = \phi$ then $f^{-1}(V) \in \mathfrak{I}$
in case $f^{-1}(V)$ is open in X
If $f^{-1}(V) \neq \phi$ then $\exists$ $x \in f^{-1}(V)$ so that $f(x) \in V$
Now if f is continuous then f is cont at x
$\Rightarrow \exists$ $H \in \mathfrak{I}$ st $x \in H$ and $f(H) \subset V$
$\Rightarrow$ $x \in H \subset f^{-1}(V)$ $H \in \mathfrak{I}$
$f^{-1}(V)$ is a nbd. of each of its points
so $f^{-1}(V)$ is $\mathfrak{I}$- open.



Converse part do urself.

Thm. A map $f: X \to Y$ is continuous iff $f^{-1}(C)$ is closed in X for every closed set $C \subset Y$.

Proof:- Let $f: X \to Y$ be a continuous map
To prove that $f^{-1}(C)$ is closed in X for each closed set $C \subset Y$.
Let $C \subset Y$ be an arbitrary closed set.
Continuity of f implies that
$$f^{-1}(Y - C) \text{ is open in } X$$
$$\Rightarrow f^{-1}(Y) - f^{-1}(C) \text{ is open in } X$$
$$\Rightarrow X - f^{-1}(C) \text{ is open in } X$$
$$\Rightarrow f^{-1}(C) \text{ is closed in } X$$

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Conversely, suppose that $f:(X,\mathfrak{J}) \to (Y,U)$ is a map such that $f^{-1}(C)$ is closed for each closed set $C \subset Y$.

To prove that f is continuous

~~By hypothesis~~

~~$f^{-1}(Y-G)$ is closed~~

Let $G \subset Y$ be an arbitrary open set then $Y-G$ is closed in Y.

By hypothesis

$$f^{-1}(Y-G) \text{ is closed in } X$$

$$f^{-1}(Y) - f^{-1}(G) \text{ '' '' '' ''}$$

$$\Rightarrow X - f^{-1}(G) \text{ is '' '' ''}$$

$$\Rightarrow f^{-1}(G) \text{ is open in } X$$

any $G \subset Y$ is open $\Rightarrow f^{-1}(G)$ is open in X which proves that f is continuous map